INTEGRATED GPS AND UWB NAVIGATION SYSTEM: (Motivates the Necessity of Non-Interference)

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ABSTRACT

We compared positioning performance of GPS and DGPS to that of GPS/DGPS integrated with non-interfering UWB transponders. Simulations of a simple test case predict 15% horizontal accuracy improvement of GPS+UWB over stand-alone GPS and 25% improvement of DGPS+UWB over pure DGPS.

1. INTRODUCTION

Recently there has been a drive to provide fast wireless radio networks [1]. Some of the proposed technology also lends itself to use for positioning [2]. Ultra-WideBand technology, in particular, works by transmitting short pulses, instead of continuously modulating a carrier signal. When it comes to navigation applications, UWB sounds compelling due to its ability to reject most multipath signals.

The Global Positioning System, GPS, provides absolute positions to users almost anywhere in the world. GPS is currently specified to have horizontal position accuracies of less than 10 meters. Several differential GPS (DGPS) services have similar accuracies in the 1-meter region.

Unchecked UWB can be harmful to GPS signal tracking [3]. We believe that such problems can be overcome with FCC-proposed emission levels, and we propose a positioning architecture that integrates GPS and UWB technology. Such a combination has the ability to improve overall positioning performance as well as augmenting position solutions in areas where GPS coverage is scarce. One could also implement a leapfrog system where sets of units would take turns stopping/moving into an area with no other navigation coverage by propagating references to last known positions.

2. THEORY

We'd like to combine measurements from both GPS receivers and UWB transponders.

2.1 Basic navigation equations

The basic measuring equations for GPS are

$$t_{user}^{(1)} = d_{user}^{(1)} + B^{(1)} - b_{user} + I_{user}^{(1)} + T_{user}^{(1)} + u_{t,user}^{(1)}$$

$$f_{user}^{(1)} = d_{user}^{(1)} + B^{(1)} - b_{user} - I_{user}^{(1)} + T_{user}^{(1)} + N_{user}^{(1)} + u_{f,user}^{(1)}$$

t and f are code phase and carrier phase measurements respectively. d is true distance between satellite (SV) and user, B is satellite clock offset and b is user clock offset. I and T are ionospheric and tropospheric delays respectively. N is a carrier cycle ambiguity term and u is a noise term. Subscripts describe who receives the signal (*user*), and superscripts signify which satellite transmitted the signal (1).

Similarly, the basic measuring equation for a pair of UWB transponders is given by

$$j_a^{(b)} = 2d_a^{(b)} + d^{(b)} + u_a^{(b)}$$

j is the roundtrip range measurement, d is true distance between transponders, d is transponder processing delay and u is a noise term. Here, subscripts signify which transponder initiated a query (a), and superscripts signify the transponder that responded to the query (b).

2.2 Linearized navigation equations

Stand-alone GPS measurements can be linearized around an estimate of current position.

$$\hat{t}_{user}^{(1)} \approx d_{user,0}^{(1)} + \frac{\partial d_{user}^{(1)}}{\partial \mathbf{x}} \Delta \mathbf{x}_{user} - b_{user}$$

$$= d_{user,0}^{(1)} - \mathbf{los}_{user}^{(1)} \cdot \Delta \mathbf{x}_{user} - b_{user}$$

$\hat{t}$ signifies code phase measurement corrected for biases in satellite clock, ionosphere and troposphere. $\mathbf{los}_{user}^{(1)}$ is the line-of-sight unit vector from the user to SV #1, and $\Delta \mathbf{x}_{user}$ is the innovation on user position from its current estimate.

We can pick one of the GPS/UWB nodes in the network to be a local reference point. In current DGPS configurations this reference point is always in some fixed location, but that is not required in cases where we're mostly interested in relative positions (e.g. formation flying).

Taking the single difference between code phase measurements of the reference node and node *a* yields

$$t_{ref}^{(1)} - t_a^{(1)} \approx \frac{\partial d_{ref}^{(1)}}{\partial \mathbf{x}} \Delta \mathbf{x}_{ref}^{(a)} + \Delta b_{ref}^{(a)} = -\mathbf{los}_{ref}^{(1)} \cdot \Delta \mathbf{x}_{ref}^{(a)} + \Delta b_{ref}^{(a)}$$

$\Delta \mathbf{x}_{ref}^{(a)}$ is the vector from the reference node to node *a* and $\Delta b_{ref}^{(a)}$ is user clock difference.

If both-way range measurements are available, we can rewrite the UWB transponder equation as

$$\frac{j_{ref}^{(a)} + j_a^{(ref)}}{4} - \frac{d^{(ref)} + d^{(a)}}{4} \approx \mathbf{los}_{ref}^{(a)} \cdot \Delta \mathbf{x}_a$$

A simple 2-D case of the integrated GPS and UWB navigation system is shown in the figure below.

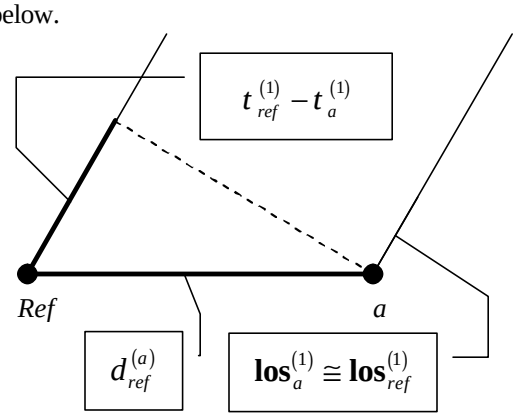


Fig. 1 Simple 2-D case

In a regular DGPS system only $t_{ref}^{(1)} - t_a^{(1)}$ would be measured, but the additional measurement of distance between nodes is provided through integration with UWB transponders.

3. COVARIANCE ANALYSIS

We would like to estimate node position accuracies as a function of total system geometry and fundamental measurement accuracies.

In the field of GPS navigation, one often splits up position error in pseudo range error multiplied by a geometry-based dilution-of-precision factor (DOP). This concept is easily extended for use with integrated GPS and UWB navigation systems.

3.1 GPS case

The full set of linearized navigation equations for code phase GPS positioning with N SVs and after correcting for SV clock errors, ionosphere and troposphere is

$$\begin{bmatrix} \hat{t}_{user}^{(1)} - d_{user,0}^{(1)} \\ \hat{t}_{user}^{(2)} - d_{user,0}^{(2)} \\ \vdots \\ \hat{t}_{user}^{(N)} - d_{user,0}^{(N)} \end{bmatrix} \approx \begin{bmatrix} -\mathbf{los}_{user}^{(1)} & -1 \\ -\mathbf{los}_{user}^{(2)} & -1 \\ \vdots & \vdots \\ -\mathbf{los}_{user}^{(N)} & -1 \end{bmatrix} \cdot \begin{bmatrix} \Delta x_{user} \\ \Delta y_{user} \\ \Delta z_{user} \\ \hline b_{user} \end{bmatrix}$$

The matrix containing line-of-sight (**los**) vectors and minus-ones is referred to as the geometry matrix, or just the G-matrix.

The above equation set is in the form $\mathbf{y} = \underset{\sim}{\mathbf{G}} \cdot \mathbf{x}$

Thus, to find the covariance matrix we perform the following operation:

$$E\left(\mathbf{y} \cdot \mathbf{y}^T\right) = E\left(\left(\underset{\sim}{\mathbf{G}} \cdot \mathbf{x}\right) \cdot \left(\underset{\sim}{\mathbf{G}} \cdot \mathbf{x}\right)^T\right)$$

After a few steps of algebra the above equation can be rewritten as

$$E\left(\mathbf{y} \cdot \mathbf{y}^T\right) = \underset{\sim}{\mathbf{G}} \cdot E\left(\mathbf{x} \cdot \mathbf{x}^T\right) \cdot \underset{\sim}{\mathbf{G}}^T$$

Yet more algebra yields

$$E\left(\mathbf{x} \cdot \mathbf{x}^T\right) =$$

$$\left(\underset{\sim}{\mathbf{G}}^T \cdot \underset{\sim}{\mathbf{G}}\right)^{-1} \underset{\sim}{\mathbf{G}}^T E\left(\mathbf{y} \cdot \mathbf{y}^T\right) \underset{\sim}{\mathbf{G}} \left(\underset{\sim}{\mathbf{G}}^T \cdot \underset{\sim}{\mathbf{G}}\right)^{-1}$$

We assume all code phase measurements are unbiased i.i.d. Gaussian with common standard deviation of σ_τ. This assumption is supported by the fact that all GPS satellites are somewhat equally far away, so the received signal power (thus, SNR) is fairly constant in most cases. The previous equation now reduces to

$$E\left(\mathbf{x} \cdot \mathbf{x}^T\right) = \left(\underset{\sim}{\mathbf{G}}^T \cdot \underset{\sim}{\mathbf{G}}\right)^{-1} \cdot s_t^2$$

In this way we find that the covariance of position is a function of satellite geometry and code phase standard deviation only.

3.2 DGPS case

The full set of linearized navigation equations for code phase DGPS positioning looks like

$$\begin{bmatrix} t_{ref}^{(1)} - t_{user}^{(1)} \\ t_{ref}^{(2)} - t_{user}^{(2)} \\ \vdots \\ t_{ref}^{(N)} - t_{user}^{(N)} \end{bmatrix} \approx \begin{bmatrix} -\mathbf{los}_{ref}^{(1)} & 1 \\ -\mathbf{los}_{ref}^{(2)} & 1 \\ \vdots & \vdots \\ -\mathbf{los}_{ref}^{(N)} & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta x_{ref}^{(user)} \\ \Delta y_{ref}^{(user)} \\ \Delta z_{ref}^{(user)} \\ \hline \Delta b_{ref}^{(user)} \end{bmatrix}$$

The user and the reference station need to have measurements from at least 4 common satellites to resolve relative user position $\{\Delta x_{ref}^{(user)}$, $\Delta y_{ref}^{(user)}$, $\Delta z_{ref}^{(user)}\}$ and clock bias $\Delta b_{ref}^{(user)}$.

The above equation set is also in the form $\mathbf{y} = \underset{\sim}{\mathbf{G}} \cdot \mathbf{x}$

To find the corresponding covariance matrix we subtract out mean values ($\tilde{\mathbf{y}} = \mathbf{y} - \overline{\mathbf{y}}$ and $\tilde{\mathbf{x}} = \mathbf{x} - \overline{\mathbf{x}}$). We assume all single difference code phase measurements are unbiased i.i.d. Gaussian with common standard deviation of $\sigma_{\tau\tau}$. After some algebra, the above equation reduces to

$$E\left(\tilde{\mathbf{x}} \cdot \tilde{\mathbf{x}}^T\right) = \left(\underset{\sim}{\mathbf{G}}^T \cdot \underset{\sim}{\mathbf{G}}\right)^{-1} \cdot s_{tt}^2$$

3.2 Integrated DGPS and UWB case

We can augment our original DGPS equation set with the actual measurements of distance between nodes in a network. A simple 3-node (*ref*, *a* and *b*) configuration has the following set of navigation equations:

$$\begin{bmatrix} t_{ref}^{(1)} - t_a^{(1)} \\ t_{ref}^{(2)} - t_a^{(2)} \\ \vdots \\ t_{ref}^{(N)} - t_a^{(N)} \\ \hline t_{ref}^{(1)} - t_b^{(1)} \\ t_{ref}^{(2)} - t_b^{(2)} \\ \vdots \\ t_{ref}^{(M)} - t_b^{(M)} \\ \hline \frac{j_{ref}^{(a)} + j_a^{(ref)}}{4} - \frac{d^{(ref)} + d^{(a)}}{4} \\ \frac{j_{ref}^{(b)} + j_b^{(ref)}}{4} - \frac{d^{(ref)} + d^{(b)}}{4} \\ \frac{j_b^{(a)} + j_a^{(b)}}{4} - \frac{d^{(b)} + d^{(a)}}{4} \end{bmatrix} \approx \left[\begin{array}{cc|cc} -\mathbf{los}_{ref}^{(1)} & -1 & \mathbf{0} & 0 \\ -\mathbf{los}_{ref}^{(2)} & -1 & \mathbf{0} & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\mathbf{los}_{ref}^{(N)} & -1 & \mathbf{0} & 0 \\ \hline \mathbf{0} & 0 & -\mathbf{los}_{ref}^{(1)} & -1 \\ \mathbf{0} & 0 & -\mathbf{los}_{ref}^{(2)} & -1 \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & 0 & -\mathbf{los}_{ref}^{(N)} & -1 \\ \hline \mathbf{los}_a^{(a)} & 0 & \mathbf{0} & 0 \\ \mathbf{0} & 0 & \mathbf{los}_{ref}^{(b)} & 0 \\ -\mathbf{los}_a^{(b)} & 0 & \mathbf{los}_a^{(b)} & 0 \end{array}\right] \cdot \begin{bmatrix} \Delta x_{ref}^{(a)} \\ \Delta y_{ref}^{(a)} \\ \Delta z_{ref}^{(a)} \\ \hline \Delta b_{ref}^{(a)} \\ \Delta x_{ref}^{(b)} \\ \Delta y_{ref}^{(b)} \\ \Delta z_{ref}^{(b)} \\ \hline \Delta b_{ref}^{(b)} \end{bmatrix}$$

The above equation set is also in the form $\mathbf{y} = \underset{\sim}{\mathbf{G}} \cdot \mathbf{x}$. We need assumptions about ranging accuracies of the UWB transponders. These accuracies should be a function of signal-to-noise ratios, transponder processing delay variabilities and the given radio channels. To a first order approximation, we'll assume that round-trip delay measurements are unbiased i.i.d Gaussian with standard deviation of σ_φ. Similarly, we assume that processing delays are unbiased i.i.d. Gaussian with standard deviations of σ_δ.

3.3 Integrated GPS and UWB case

Whereas the above DGPS integration may improve positioning relative to a reference node, this node may well have a biased absolute position due to e.g. ionospheric effects or multipath. Thus, the entire network would have biased absolute positions. However, we can reformulate our navigation equations for absolute positioning as well.

The left-hand side of the 3-node case is shown below (after using standard code phase corrections for satellite clock, ionosphere and troposphere):

$$\begin{bmatrix} \tilde{t}_a^{(1)} \\ \vdots \\ \tilde{t}_a^{(N)} \\ \hline \tilde{t}_b^{(1)} \\ \vdots \\ \tilde{t}_b^{(N)} \\ \hline \tilde{t}_c^{(1)} \\ \vdots \\ \tilde{t}_c^{(N)} \\ \hline \frac{j_b^{(a)} + j_a^{(b)}}{4} - \frac{d^{(b)} + d^{(a)}}{4} \\ \frac{j_c^{(a)} + j_a^{(c)}}{4} - \frac{d^{(c)} + d^{(a)}}{4} \\ \frac{j_b^{(c)} + j_c^{(b)}}{4} - \frac{d^{(b)} + d^{(c)}}{4} \end{bmatrix} - \begin{bmatrix} \mathbf{los}_a^{(1)} & \cdots & \mathbf{0} & -\mathbf{los}_a^{(1)} & \mathbf{0} & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{los}_a^{(N)} & -\mathbf{los}_a^{(N)} & \mathbf{0} & \mathbf{0} \\ \mathbf{los}_b^{(1)} & \cdots & \mathbf{0} & \mathbf{0} & -\mathbf{los}_b^{(1)} & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{los}_b^{(N)} & \mathbf{0} & -\mathbf{los}_b^{(N)} & \mathbf{0} \\ \mathbf{los}_c^{(1)} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{los}_c^{(1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{los}_c^{(N)} & \mathbf{0} & \mathbf{0} & -\mathbf{los}_c^{(N)} \\ \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{los}_a^{(b)} & \mathbf{los}_a^{(b)} & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{los}_a^{(c)} & \mathbf{0} & \mathbf{los}_a^{(c)} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & -\mathbf{los}_b^{(c)} & \mathbf{los}_b^{(c)} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x}^{(1)} \\ \vdots \\ \mathbf{x}^{(N)} \\ \mathbf{x}_{a,0} \\ \mathbf{x}_{b,0} \\ \mathbf{x}_{c,0} \end{bmatrix}$$

In the above, $\mathbf{x}^{(1)}$ is the 3-D position of GPS satellite #1 in Earth-Centered Earth-Fixed (ECEF) coordinates, and $\mathbf{x}_{a,0}$ is our current estimates of position for node *a*. The above set is equal to

$$\begin{bmatrix} -\mathbf{los}_a^{(1)} & -1 & \mathbf{0} & 0 & \mathbf{0} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\mathbf{los}_a^{(N)} & -1 & \mathbf{0} & & \mathbf{0} & \\ \hline \mathbf{0} & 0 & -\mathbf{los}_b^{(1)} & -1 & \mathbf{0} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & 0 & -\mathbf{los}_b^{(N)} & -1 & \mathbf{0} & 0 \\ \hline \mathbf{0} & 0 & \mathbf{0} & 0 & -\mathbf{los}_c^{(1)} & -1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & 0 & \mathbf{0} & 0 & -\mathbf{los}_c^{(N)} & -1 \\ \hline -\mathbf{los}_a^{(b)} & 0 & \mathbf{los}_a^{(b)} & 0 & \mathbf{0} & 0 \\ -\mathbf{los}_a^{(c)} & 0 & \mathbf{0} & 0 & \mathbf{los}_a^{(c)} & 0 \\ \mathbf{0} & 0 & -\mathbf{los}_b^{(c)} & 0 & \mathbf{los}_b^{(c)} & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta\mathbf{x}_a \\ \hline b_a \\ \hline \Delta\mathbf{x}_b \\ \hline b_b \\ \hline \Delta\mathbf{x}_c \\ \hline b_c \end{bmatrix}$$

Here, $\Delta\mathbf{x}_a$, $\Delta\mathbf{x}_b$ and $\Delta\mathbf{x}_c$ are innovations on the 3-D positions of nodes *a*, *b* and *c*. The *b*-s represent user clock biases of the same three nodes.

The equation set for integrated GPS and UWB transponders is on the form

$$\mathbf{y} - \underset{\sim}{\mathbf{G}}_1 \cdot \mathbf{x}_0 = \underset{\sim}{\mathbf{G}}_2 \cdot \mathbf{x}$$

Both geometry matrices and $\mathbf{x}_0$ are deterministic for a given SV-user geometry. Thus, the variability of positions and user clock biases can be expressed as

$$E\left(\mathbf{x}\cdot\mathbf{x}^T\right)=$$

$$\left(\underset{\sim}{\mathbf{G}}_2^T\cdot\underset{\sim}{\mathbf{G}}_2\right)^{-1}\underset{\sim}{\mathbf{G}}_2^T E\left(\mathbf{y}\cdot\mathbf{y}^T\right)\underset{\sim}{\mathbf{G}}_2\left(\underset{\sim}{\mathbf{G}}_2^T\cdot\underset{\sim}{\mathbf{G}}_2\right)^{-1}-$$

$$\left(\underset{\sim}{\mathbf{G}}_2^T\cdot\underset{\sim}{\mathbf{G}}_2\right)^{-1}\underset{\sim}{\mathbf{G}}_2^T\underset{\sim}{\mathbf{G}}_1\mathbf{x}_0\mathbf{x}_0^T\underset{\sim}{\mathbf{G}}_1^T\underset{\sim}{\mathbf{G}}_2\left(\underset{\sim}{\mathbf{G}}_2^T\cdot\underset{\sim}{\mathbf{G}}_2\right)^{-1}$$

$E\left(\mathbf{y}\cdot\mathbf{y}^T\right)$ is now a function of the statistical properties of code phase measurements, user clocks, UWB round-trip measurements and transponder processing delay. Unlike the pure DGPS case, we can't simplify the above equation much.

4. SIMULATIONS

We'd like to compare predicted positioning performance of pure GPS vs. GPS integrated with UWB transponders. For all our simulations, we'll pick a satellite geometry where one SV is directly overhead, and three other SVs have elevation angles of 10 degrees and are spread out 120 degrees apart in azimuth starting from true north. The figure below shows our fixed SV geometry.

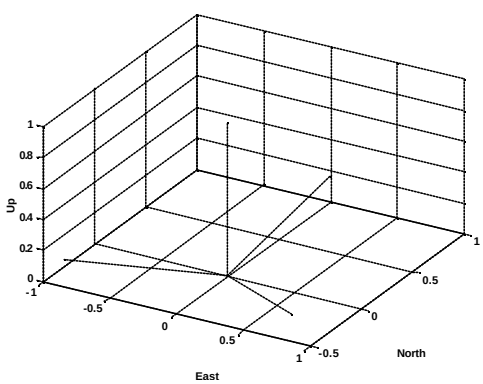


Fig. 2 SV geometry

For integrated GPS and UWB, we will pick a 3-node case where the first node is located at the origin of a local coordinate system and the second node is located 200 meters in the local ENU x-direction. The third node will be swept in a 200-meter circular arc on the local xy-plane, starting at the location of the second node. The next figure shows the node setup.

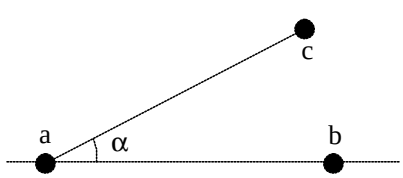


Fig. 3 Node geometry

In the following we'll assume all code phase measurements are unbiased i.i.d. Gaussian having standard deviations of 4 meters, σ_τ. Single differences we'll assume have standard deviations of .4 meters, $\sigma_{\tau\tau}$ (reduce many common mode errors like ionosphere). Based on previous UWB tests [4], we conservatively estimate standard deviations of .1 meters both for UWB round-trip ranging and transponder processing delays (all i.i.d. Gaussian).

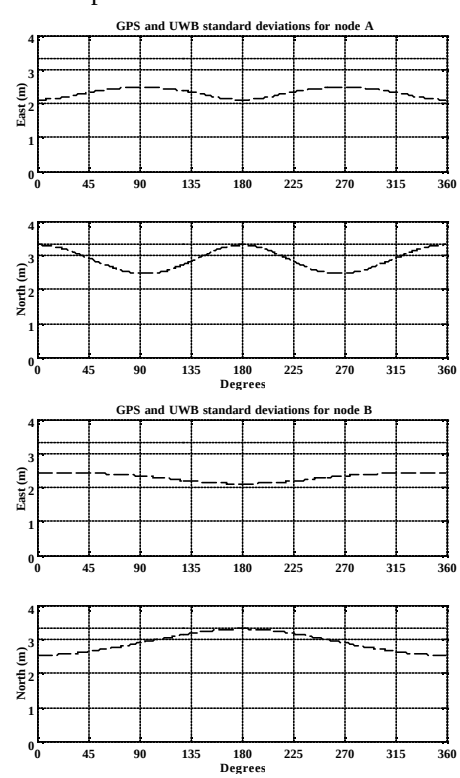
4.1 GPS vs. GPS+UWB

Since GPS SVs are on the order of 20,000 km away from the Earth, GPS receivers that are only 200 m apart will have the same predicted positioning performance under the assumptions above. The table below shows standard deviations of user position and clock bias (in meters).

σ_{East}	σ_{North}	σ_{Up}	σ_b
3.3164	3.3164	5.5894	2.9184

The UWB transponder system doesn't measure user clock biases, and all nodes are in the xy-plane. Thus, σ_{Up} and σ_b will remain unchanged in the configuration at hand.

The next plots show standard deviations in East and North directions for all three nodes both for stand-alone GPS and for GPS integrated with UWB transponders.



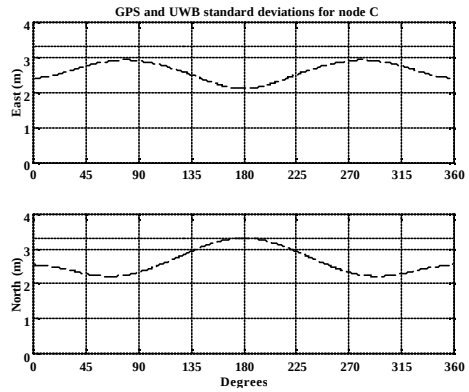


Fig. 4 GPS and GPS+UWB

In Figures 4 and 5, solid lines mean GPS-only and dashed lines signify GPS integrated with UWB.

4.2 DGPS vs. DGPS+UWB

Using the same argument as in Chapter 4.1, we calculate standard deviations and clock biases for nodes *b* and *c* relative to node *a*.

σ_{East}	σ_{North}	σ_{Up}	σ_b
.3316	.3316	.5589	.2918

Again, σ_{Up} and σ_b will remain unchanged in our integration with UWB.

The plots below show how expected relative position accuracies in East and North directions change with angle α for nodes *b* and *c*.

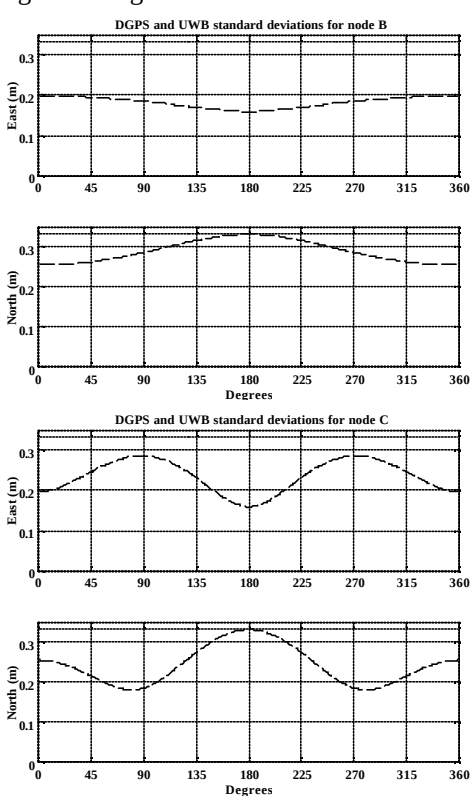


Fig. 5 DGPS and DGPS+UWB

5. CONCLUSION

Although we worked a somewhat limited case, we believe that integrating GPS with UWB transponders has great potential for improving navigation performance. Our simulations showed an average improvement of horizontal position accuracy of 15% vs. stand-alone GPS and 25% vs. DGPS. Vertical position accuracy would also improve if additional nodes were added outside the xy-plane.

Not only do our simulations show a significant improvement of position accuracy, but the additional information provided by UWB transponders may also improve total system availability. An ordinary GPS receiver needs at least 4 SVs in view to resolve its position and clock bias. However, a GPS receiver integrated with a UWB transponder may require less than 4 SVs if other nodes in a network provide that user with other ranging information. In other words, the addition of UWB transponders makes the original navigation equations over-determined.

6. FUTURE WORK

Our scenario had a fixed GPS geometry, so incorporating the real GPS constellation should bring our simulations closer to the truth.

Furthermore, we choose to use a fairly simple model for the UWB transponder. Using double round-trip measurements and having all nodes broadcast their current clock states would enable synchronization of the entire network [4]. Such a scheme would also enable measurement of user clock offsets. This could be used to further improve clock bias estimates.

Whereas we only considered GPS code phase measurements, the advent of carrier phase information could improve positioning accuracies even more.

7. REFERENCES

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